

## Multiparticle correlations and momentum conservation in nucleus-nucleus collisions

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Particle correlations are very actively studied in heavy-ion collisions at ultra-relativistic energies. Here, an attempt is made at determining a proper reference for such studies, by taking properly into account the multiparticle correlations induced by the conservation of total momentum in the collisions.

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## 1. Introduction

High-energy collisions of heavy nuclei result in the emission of up to several thousands of particles. Such large numbers reflect the complexity of the collisions; yet, they allow for a wealth of different tools to characterize the processes that take place, in particular to assess the properties of the medium that is created. Among the possible approaches, the presence of these many particles calls in a natural way for statistical descriptions, as well as for observables involving several particles. Various studies are thus devoted to extracting correlations between pairs, triplets, and more generally  $n$ -tuples of particles.

The purposes of these investigations are manifold, as are the underlying physics pictures: To mention only correlations in momentum space, there are studies focusing on identical particles that are close in momentum (and in position) space, searching for evidence of (anti)symmetrization of the corresponding wave-function [1]. Other, closely related studies deal with pairs of non-identical particles with close velocities, to estimate the difference between their emission times [2]. Another phenomenon investigated through studies of the correlations between emitted particles is the anisotropy in the transverse plane of the emission pattern induced by the finite impact parameter between the nuclei (“anisotropic flow”) [3]. The extended transverse-momentum range that became available with collisions at RHIC allowed novel investigations, involving particles with high transverse momenta  $p_T$ , with a view to studying jets. A first kind of such study is that of correlations in azimuth between pairs of particles with both high transverse momenta [4], to look for the presence of structures, close or away in azimuth to a high- $p_T$  reference (“trigger”) particle, that resemble the jets seen in  $pp$  collisions. The next step consists of studies of correlations between a high- $p_T$  trigger and low- $p_T$  “associated” particles [5], so as to characterize the response of the created medium to the propagation of a high- $p_T$  parton (or, less plausibly, hadron).

Whatever the specific aim of a correlation study, it boils down to a simple principle, namely to try to identify the trace of some genuine dynamical effect in the joint two-, three-,  $M$ -particle distributions. To accomplish this, one needs to determine properly what the expectation would be for these distributions in the absence of such a dynamical effect. Now, such a “no-dynamics”  $M$ -particle distribution is not merely the product of  $M$  single-particle distributions, for there exists a trivial correlation between arbitrary final-state particles, due to the conservation of total momentum, which imposes some constraints on the joint distributions [6]. The computation of these constraints will be discussed in section 3, using the general formalism from probability theory introduced in section 2. In section 4, I shall further discuss the meaning of this ever-present correlation due to global momentum conservation in the collision, and speculate on possible ways to take it into account in correlation studies.

## 2. Probability distributions and cumulants

Consider a collision with a total of  $N$  particles in the final state<sup>1</sup> (throughout this paper,  $N$  is assumed to be large). The basic observable in studies of the momentum correlations between  $M$  particles among the  $N$  is the joint distribution  $d^M \mathcal{N} / d\mathbf{p}_1 \cdots d\mathbf{p}_M$ . To avoid normalization issues, it

<sup>1</sup> $N$  includes *all* particles, including the non-detected ones.

is more convenient to consider the joint  $M$ -particle *probability* distribution  $f(\mathbf{p}_1, \dots, \mathbf{p}_M)$ , which is by definition normalized to unity, and therefore (roughly) independent of the system size.

By definition,  $M$  particles with momenta  $\mathbf{p}_1, \dots, \mathbf{p}_M$  are statistically independent from each other if and only if the corresponding joint probability distribution can be factorized into the product of the  $M$  single-particle probability distributions:  $f(\mathbf{p}_1, \dots, \mathbf{p}_M) = f(\mathbf{p}_1) \cdots f(\mathbf{p}_M)$ . Reciprocally, if they are not independent, this factorization no longer holds and the joint probability distribution involves further terms: the joint probability distribution can be expanded into a sum over all products of *cumulants* corresponding to distinct partitions of the  $M$  particles. For instance, at the two- and three-particle levels:

$$f(\mathbf{p}_1, \mathbf{p}_2) = f_c(\mathbf{p}_1)f_c(\mathbf{p}_2) + f_c(\mathbf{p}_1, \mathbf{p}_2), \quad (2.1)$$

$$\begin{aligned} f(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = & f_c(\mathbf{p}_1)f_c(\mathbf{p}_2)f_c(\mathbf{p}_3) + f_c(\mathbf{p}_1)f_c(\mathbf{p}_2, \mathbf{p}_3) + f_c(\mathbf{p}_2)f_c(\mathbf{p}_1, \mathbf{p}_3) + f_c(\mathbf{p}_3)f_c(\mathbf{p}_1, \mathbf{p}_2) \\ & + f_c(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3), \end{aligned} \quad (2.2)$$

where the single-particle cumulant is equal to the single-particle probability function,  $f_c(\mathbf{p}) = f(\mathbf{p})$ . The  $M$ -particle cumulant  $f_c(\mathbf{p}_1, \dots, \mathbf{p}_M)$  corresponds to the “genuine” correlation between the  $M$  particles. The physical interpretation of the cumulant expansion is straightforward: the joint  $M$ -particle distribution depends not only on the genuine  $M$ -particle correlation, but also on all the possible correlations involving subsets among the  $M$  particles. As an example, think of two particles emitted exactly back-to-back with  $\mathbf{p}_2 = -\mathbf{p}_1$  (as the two pions from a decaying  $\rho$  meson in the rest frame of the latter). The two-particle probability distribution reads  $f(\mathbf{p}_1, \mathbf{p}_2) = f(\mathbf{p}_1)\delta(\mathbf{p}_1 + \mathbf{p}_2)$  (where, in the  $\rho \rightarrow \pi\pi$  case, the single-particle distribution  $f(\mathbf{p}_1)$  actually reduces to the angular distribution since  $|\mathbf{p}_1|$  is also fixed), while the corresponding cumulant is obviously non-vanishing, see equation (2.1). Other illustrations of the difference between the distributions  $f$  and the cumulants  $f_c$  can be found in reference [7].

While equations (2.1)-(2.2), and so on, can be inverted one after the other to yield the cumulants as functions of the joint probability distributions, there is a more systematic way to perform the same operation. First, one defines a generating function of the joint multiparticle probability distributions:

$$G(x_1, \dots, x_N) \equiv 1 + x_1 f(\mathbf{p}_1) + x_2 f(\mathbf{p}_2) + \cdots + x_1 x_2 f(\mathbf{p}_1, \mathbf{p}_2) + \cdots, \quad (2.3)$$

and similarly for every order  $M$ , where  $x_1, \dots, x_N$  are auxiliary variables. Given this generating function of the joint probability distributions, the function that generates the cumulants is simply its logarithm [8]:

$$\ln G(x_1, \dots, x_N) \equiv x_1 f_c(\mathbf{p}_1) + x_2 f_c(\mathbf{p}_2) + \cdots + x_1 x_2 f_c(\mathbf{p}_1, \mathbf{p}_2) + \cdots. \quad (2.4)$$

Thus, the knowledge of one of these functions automatically translates into that of the other. In the following section, I shall use this property to sketch the computation of the multiparticle cumulants due to the momentum-conservation constraint, starting from the expression of the joint probability distribution for  $M$  particles. Moreover, I shall also use “scaled” cumulants  $\tilde{f}_c(\mathbf{p}_1, \dots, \mathbf{p}_M) \equiv f_c(\mathbf{p}_1, \dots, \mathbf{p}_M) / [f(\mathbf{p}_1) \cdots f(\mathbf{p}_M)]$ .

### 3. Multiparticle cumulants from momentum conservation

As stated in the introduction, the purpose is to determine how global momentum conservation affects the joint multiparticle probability distributions of final-state particles in a large-multiplicity event like a heavy-ion collision. That is, given  $N$  particles with momenta  $\mathbf{p}_1, \dots, \mathbf{p}_N$  obeying the constraint  $\mathbf{p}_1 + \dots + \mathbf{p}_N = \mathbf{0}$ , what is the resulting  $M$ -particle cumulant? This question was to my knowledge first addressed (albeit semi-quantitatively) in the two-particle case in reference [6]. A quantitative estimate of the two-particle correlation was then derived in the frame of some anisotropic-flow measurement [9], then independently rediscovered in the same context [10]. In both these cases, the computation relied on the use of the central-limit theorem for the distribution of the sum of  $n \gg 1$  (yet  $n < N$ ) *uncorrelated* momenta. Eventually, a general approach to compute the cumulants to arbitrary order was introduced in reference [11], making use of the generating functions of joint probability distributions and of cumulants and of a saddle-point integration.

The starting point of these calculations is the expression of the joint  $M$ -particle probability distribution under the momentum-conservation constraint:

$$f(\mathbf{p}_1, \dots, \mathbf{p}_M) \equiv \frac{\left( \prod_{j=1}^M F(\mathbf{p}_j) \right) \int \delta(\mathbf{p}_1 + \dots + \mathbf{p}_N) \prod_{j=M+1}^N [F(\mathbf{p}_j) d\mathbf{p}_j]}{\int \delta(\mathbf{p}_1 + \dots + \mathbf{p}_N) \prod_{j=1}^N [F(\mathbf{p}_j) d\mathbf{p}_j]}, \quad (3.1)$$

where  $F(\mathbf{p})$  is the single-particle probability distribution “unrenormalized” for the momentum conservation constraint: to leading order in  $1/N$ , it equals the measured single-particle probability distribution  $f(\mathbf{p})$ , yet they actually differ at the next-to-leading order [10]. In the previous calculations [9–11],  $F$  was assumed to be isotropic; here we shall relax this assumption and consider the more realistic case of collisions with anisotropic expansion (flow) in the transverse plane.<sup>2</sup> Quite obviously, the Dirac  $\delta$  in equation (3.1) represents the constraint from global momentum conservation: in its absence,  $F(\mathbf{p}) = f(\mathbf{p})$  and the joint probability distribution would simply factorize into the product  $F(\mathbf{p}_1) \dots F(\mathbf{p}_M)$ .

To derive the cumulants arising from global momentum conservation, the most convenient and systematic way is to introduce equation (3.1) in the expression of the generating function (2.3) so as to compute the latter [11]. After introducing a Fourier representation of the Dirac distribution, one finds that  $G(x_1, \dots, x_N)$  can be expressed as the integral over the Fourier conjugate variable  $\mathbf{k}$  of the exponential of a function  $N\mathcal{F}(\mathbf{k})$ , which also depends on the auxiliary variables  $x_j$ .<sup>3</sup> Since  $N$  is supposed to be large, this integral can be performed by a saddle-point approximation, provided one finds the position  $\mathbf{k}_0$  of the maximum. The key to obtaining the successive cumulants is then first to solve to a given order in  $\bar{x}/N$  the equation giving  $\mathbf{k}_0$  (a solution to the order  $M-1$  is required for

<sup>2</sup>The recipe for extending the calculation to the non-isotropic case was briefly given in reference [11], and the corresponding expression for two-particle correlations can be found in reference [12].

<sup>3</sup>More precisely,  $\mathcal{F}$  only depends on  $x_j$  through combinations  $x_j F(p_j)/N$ : this allows one to replace  $x_j$  in  $G$  by  $\bar{x}_j \equiv x_j F(p_j)$  — which amounts, to leading order in  $1/N$ , to replacing the cumulants  $f_c$  by the scaled cumulants  $\bar{f}_c$  — and to derive the scaling with  $N$  of the cumulants [11]: the  $M$ -particle cumulant scales as  $1/N^{M-1}$ .

the  $M$ -particle cumulant):

$$\left( \sum_{j=1}^N \frac{\bar{x}_j}{N} \frac{e^{i\mathbf{k}_0 \cdot \mathbf{p}_j}}{\langle e^{i\mathbf{k}_0 \cdot \mathbf{p}} \rangle} - 1 \right) \langle \mathbf{p} e^{i\mathbf{k}_0 \cdot \mathbf{p}} \rangle = \sum_{j=1}^N \frac{\bar{x}_j}{N} \mathbf{p}_j e^{i\mathbf{k}_0 \cdot \mathbf{p}_j}, \quad (3.2)$$

where the angular brackets denotes a  $F(\mathbf{p})$ -weighted average. Then, one uses this expression of  $\mathbf{k}_0$  to compute the value of  $N\mathcal{F}(\mathbf{k}_0)$ :

$$N\mathcal{F}(\mathbf{k}_0) \equiv N \left( \ln \langle e^{i\mathbf{k}_0 \cdot \mathbf{p}} \rangle + \sum_{j=1}^N \frac{\bar{x}_j}{N} \frac{e^{i\mathbf{k}_0 \cdot \mathbf{p}_j}}{\langle e^{i\mathbf{k}_0 \cdot \mathbf{p}} \rangle} \right). \quad (3.3)$$

The coefficient of  $\bar{x}_{j_1} \dots \bar{x}_{j_M}$  in  $N\mathcal{F}(\mathbf{k}_0)$  is then the scaled cumulant  $\bar{f}_c(\mathbf{p}_1, \dots, \mathbf{p}_M)$ . For instance, one finds (we assume for the sake of brevity that  $\langle \mathbf{p} \rangle = \mathbf{0}$ , otherwise one only need replace  $\mathbf{p}$  by  $\mathbf{p}' \equiv \mathbf{p} - \langle \mathbf{p} \rangle$  in the following formulas)

$$\bar{f}_c(\mathbf{p}_1, \mathbf{p}_2) = -\frac{p_{1,x}p_{2,x}}{N\langle p_x^2 \rangle} - \frac{p_{1,y}p_{2,y}}{N\langle p_y^2 \rangle} - \frac{p_{1,z}p_{2,z}}{N\langle p_z^2 \rangle}, \quad (3.4)$$

$$\begin{aligned} \bar{f}_c(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = & - \sum_{1 \leq j < k \leq 3} \left( \frac{p_{j,x}p_{k,x}}{N^2\langle p_x^2 \rangle} + \frac{p_{j,y}p_{k,y}}{N^2\langle p_y^2 \rangle} + \frac{p_{j,z}p_{k,z}}{N^2\langle p_z^2 \rangle} \right) \\ & + \left( \frac{p_{1,x}p_{2,x}}{N^2\langle p_x^2 \rangle} + \frac{p_{1,y}p_{2,y}}{N^2\langle p_y^2 \rangle} + \frac{p_{1,z}p_{2,z}}{N^2\langle p_z^2 \rangle} \right) \left( \frac{p_{1,x}p_{3,x}}{N^2\langle p_x^2 \rangle} + \frac{p_{1,y}p_{3,y}}{N^2\langle p_y^2 \rangle} + \frac{p_{1,z}p_{3,z}}{N^2\langle p_z^2 \rangle} \right) \\ & + \left( \frac{p_{1,x}p_{2,x}}{N^2\langle p_x^2 \rangle} + \frac{p_{1,y}p_{2,y}}{N^2\langle p_y^2 \rangle} + \frac{p_{1,z}p_{2,z}}{N^2\langle p_z^2 \rangle} \right) \left( \frac{p_{2,x}p_{3,x}}{N^2\langle p_x^2 \rangle} + \frac{p_{2,y}p_{3,y}}{N^2\langle p_y^2 \rangle} + \frac{p_{2,z}p_{3,z}}{N^2\langle p_z^2 \rangle} \right) \\ & + \left( \frac{p_{1,x}p_{3,x}}{N^2\langle p_x^2 \rangle} + \frac{p_{1,y}p_{3,y}}{N^2\langle p_y^2 \rangle} + \frac{p_{1,z}p_{3,z}}{N^2\langle p_z^2 \rangle} \right) \left( \frac{p_{2,x}p_{3,x}}{N^2\langle p_x^2 \rangle} + \frac{p_{2,y}p_{3,y}}{N^2\langle p_y^2 \rangle} + \frac{p_{2,z}p_{3,z}}{N^2\langle p_z^2 \rangle} \right). \quad (3.5) \end{aligned}$$

In these expressions, the  $x$ -,  $y$ - and  $z$ -directions are the principal axes that diagonalize the tensor  $\langle \mathbf{p} \otimes \mathbf{p} \rangle$ : in practice, one axis will lie along the beam direction, one along the impact parameter of the nucleus-nucleus collision, which is the only preferred direction in the transverse plane, and the third one will be perpendicular to the other two. If  $F(\mathbf{p})$  is isotropic,  $\langle p_x^2 \rangle = \langle p_y^2 \rangle = \langle p_z^2 \rangle$ , so that one recovers the formulas given in reference [11].

In nucleus-nucleus collisions at ultra-relativistic energies, the mean square momentum along the beam direction ( $z$ ) is typically significantly larger than the mean square transverse components. As a consequence, the terms involving the  $z$ -component in the expressions of the cumulants are much smaller than those involving the other two components, especially when one considers final-state particles emitted close to mid-rapidity. Therefore, I shall drop these longitudinal-momentum terms from now on, which amounts to considering only the constraint imposed by the conservation of total transverse momentum  $\mathbf{p}_T$ .

#### 4. Defining a minimally-biased background for correlation studies

The behaviour of the three-particle cumulant resulting from the momentum-conservation constraint, equation (3.5), was discussed in some detail in reference [13] in the case of an isotropic transverse emission of particles. Here, I shall now discuss further the expression of the two-particle

cumulant and its content. The physical meaning of equation (3.4) is intuitive: momentum conservation induces an “anti-correlation” between the momenta of final-state particles. This is of course reminiscent of the situation in which only two particles are emitted, with back-to-back equal momenta (note, however, that the calculation reported here cannot cover that case, since it assumes  $N \gg 1$ ). Additionally, the anti-correlation is largest between particles with larger momenta; it is also largest in events with a smaller total multiplicity  $N$ .

Let me emphasize the implications of the expression of the two-particle cumulant. In collisions in which  $N$  particles are in the final state, equation (3.4) implies that the joint two-particle probability distribution reads (assuming first that particles are emitted isotropically, which is a fair approximation in the transverse plane for central collisions, so that  $\langle p_x^2 \rangle = \langle p_y^2 \rangle = \langle p_T^2 \rangle / 2$ ):

$$f(\mathbf{p}_{T1}, \mathbf{p}_{T2}) = f(\mathbf{p}_{T1})f(\mathbf{p}_{T2}) \left( 1 - \frac{2\mathbf{p}_{T1} \cdot \mathbf{p}_{T2}}{N\langle p_T^2 \rangle} \right). \quad (4.1)$$

In other words, although the particle emission is isotropic, i.e.  $f(\mathbf{p}_T)$  depends on  $p_T$  only, yet given a “trigger” particle with transverse momentum  $\mathbf{p}_{T1}$ , the *conditional* probability to find an “associated” particle with transverse momentum  $\mathbf{p}_{T2}$  is not isotropic:

$$f(\mathbf{p}_{T2}|\mathbf{p}_{T1}) = \frac{f(\mathbf{p}_{T1}, \mathbf{p}_{T2})}{f(\mathbf{p}_{T1})} = f(\mathbf{p}_{T2}) \left( 1 - \frac{2\mathbf{p}_{T1} \cdot \mathbf{p}_{T2}}{N\langle p_T^2 \rangle} \right) \neq f(\mathbf{p}_{T2}). \quad (4.2)$$

Thus, the conservation of transverse momentum induces a sinusoidal modulation of the probability distribution of “associated” particles, with a minimum of the distribution along the trigger-particle momentum: there is a larger probability that the momentum of the associated particle points in the hemisphere opposite to that of the trigger particle. The amplitude of the modulation increases with the values of both the trigger momentum  $p_{T1}$  and the associated momentum  $p_{T2}$ , and it decreases with increasing  $N$ .

One recognizes in equation (4.2) the difference between *marginal* [ $f(\mathbf{p}_{T2})$ ] and *conditional* [ $f(\mathbf{p}_{T2}|\mathbf{p}_{T1})$ ] probability distributions [8]. Since the former is — up to a normalization factor — the measured single-particle distribution, it is most tempting to use it in studies of two-particle correlations; yet one must rather use the latter once a first particle momentum  $p_{T1}$  in the event is fixed as a reference. This is not to be unexpected: by removing from the event this trigger particle, one obtains a collection of final-state particles (including the non-measured ones) which is *not* a “valid” event, since the sum of the transverse momenta of the particles does not vanish. Hence there is no reason why the single-particle probability distribution for this collection should be the same as for real events satisfying momentum conservation.<sup>4</sup> Thus, in studies of two-particle correlations in which the transverse momentum  $\mathbf{p}_{T1}$  of one of the particles has been fixed — thereby implicitly selecting a subset of the whole available sample of events — one should consider the conditional probability distribution  $f(\mathbf{p}_{T2}|\mathbf{p}_{T1})$  of equation (4.2), including the momentum-conservation constraint, as the proper “reference” distribution of associated-particle momenta  $\mathbf{p}_{T2}$ , over which

<sup>4</sup>In fact, from the mathematical point of view, these collections of  $N - 1$  particles with non-vanishing total transverse momentum  $\sum \mathbf{p}_T = -\mathbf{p}_{T1}$  are equivalent to events with a mean first-harmonic anisotropic-flow component (directed flow)  $\bar{v}_1 = -p_{T1}/[(N-1)\langle p_T \rangle]$ , the direction of the equivalent reaction plane being that of  $\mathbf{p}_{T1}$ . Therefore, the corresponding single-particle distribution includes a  $2v_1 \cos(\varphi - \varphi_1)$  azimuthal modulation, which is absent from the original single-particle distribution of the real events, which we assumed isotropic.

dynamical effects are to be investigated. This is admittedly slightly unsatisfactory, since the conditional probability distribution is not measured, and depends on two unknown quantities: the total number  $N$  of final-state particles and the mean square transverse momentum  $\langle \mathbf{p}_T^2 \rangle$  (albeit only through their product).<sup>5</sup> Yet they can be estimated, as was done to take into account the conservation of total momentum in analyses of anisotropic flow [10, 14] or in femtoscopy studies [12]. Only at this price can one determine the “minimally-biased background” which is what the distribution of associated particles would look like in the absence of non-trivial correlations. For Au-Au collisions at RHIC energies, the corresponding correction might be of at most 1 or 2% when choosing a high- $p_T$  trigger particle; yet this is of the same relative magnitude as the effects that are measured [5], so that such a precision is necessary if one wants to establish the existence of specific dynamical effects and to quantify their importance.

Let me now comment on the two-particle cumulants in the more general case where particles are not emitted isotropically in the plane transverse to the beam. More precisely, I shall consider particles emitted with a mean second-harmonic transverse anisotropy (elliptic flow)  $\bar{v}_2$  defined by  $\bar{v}_2 \equiv \langle p_x^2 - p_y^2 \rangle / \langle p_x^2 + p_y^2 \rangle$ .<sup>6</sup> This definition yields at once the identities  $\langle p_x^2 \rangle = (1 + \bar{v}_2) \langle p_T^2 \rangle / 2$  and  $\langle p_y^2 \rangle = (1 - \bar{v}_2) \langle p_T^2 \rangle / 2$ , which one can insert in equation (3.4):

$$\bar{f}_c(\mathbf{p}_{T1}, \mathbf{p}_{T2}) = -\frac{2}{N \langle p_T^2 \rangle} \left( \frac{p_{1,x} p_{2,x}}{1 + \bar{v}_2} + \frac{p_{1,y} p_{2,y}}{1 - \bar{v}_2} \right), \quad (4.3)$$

from which one deduces the conditional probability distribution  $f(\mathbf{p}_{T2} | \mathbf{p}_{T1})$  of particles associated to a trigger particle with transverse momentum  $\mathbf{p}_{T1}$ . One sees that the effect of the constraint from momentum conservation is larger in the  $y$ -direction, i.e. perpendicular to the nucleus-nucleus impact parameter, than in the  $x$ -direction. That is quite normal, since particles with a transverse momentum along  $y$  are less numerous than those pointing in the  $x$ -direction (remember that values of  $v_2(p_T) = 0.16$  were reported for  $p_T \geq 2$  GeV/ $c$  in minimum-bias Au-Au collisions at RHIC [15], meaning that twice more charged hadrons are emitted in the  $x$ -direction than perpendicular to it). As a consequence, if the trigger particle is chosen with  $\mathbf{p}_{T1}$  in the  $y$ -direction, only few particles are present to balance its momentum, so that those few ones are more strongly correlated to it than if we were considering a trigger particle with a momentum in the  $x$ -direction.

The dependence of the strength of the two-particle cumulant (4.3) on the azimuths of the two particles means that the correction for the momentum-conservation effect has to be performed with some care. Let me illustrate that with an example. I have already argued above why one should use the conditional probability distribution  $f(\mathbf{p}_{T2} | \mathbf{p}_{T1})$  instead of the marginal one when investigating the possible structures associated with the presence of a (high- $p_T$ ) trigger particle. If only the average correction, corresponding to the isotropic case (4.1), were used instead of the azimuthally-dependent one (4.3), this would yield an over-correction (resp. an under-correction) of  $f(\mathbf{p}_{T2} | \mathbf{p}_{T1})$  for trigger particles emitted along the nucleus-nucleus impact parameter (resp. emitted along the  $y$ -axis). As a result, one would observe a small (using typical RHIC values for  $N$ ,  $\langle p_T^2 \rangle$  and  $\bar{v}_2$ , of order

<sup>5</sup>Furthermore, the averages  $\langle \dots \rangle$  involve the non-measurable distribution “in the absence of momentum conservation”  $F(\mathbf{p})$ , rather than the physical distribution  $f(\mathbf{p})$  [12]. This is however but a minor issue, as the difference between  $F$  and  $f$  is of subleading order in  $1/N$ , while here we only consider leading-order quantities.

<sup>6</sup>This definition yields values of  $\bar{v}_2$  that are typically a factor 2 larger than those obtained with the more conventional definition  $v_2 \equiv \langle (p_x^2 - p_y^2) / (p_x^2 + p_y^2) \rangle$ , so that at RHIC energies  $\bar{v}_2 \approx 0.1$  in mid-central Au-Au collisions.

$\lesssim 0.005$ ) spurious bump (resp. dip) at  $180^\circ$  away from the trigger in the conditional probability distribution for  $\mathbf{p}_{T2}$ . This azimuthally-dependent spurious structure due to an inaccurate definition of the “minimally-biased background”, which is to be the reference over which correlations of dynamical origin can be observed, would prevent any accurate determination of these interesting correlations, by mimicking irrelevant features. Further implications of the azimuthal dependence of the two-particle cumulant (4.3) due to momentum conservation are discussed in reference [16].

In summary, I have recalled that the general purpose of studying correlations is to yield evidence of phenomena that go beyond trivial expectations. In the specific context of high-energy collisions, correlations between any number of final-state particles are induced by the conservation of total momentum, which are not of dynamical origin. These uninteresting correlations can be computed, thereby allowing one to define a “minimally-biased background”, including the effect of total-momentum conservation, which is viewed as the reference over which genuine dynamical effects might be revealed.

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